Reg. No. : | | | | | | | | | | | |

Question Paper Code : 80870

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

Fourth Semester

Computer Science and Engineering

MA 8402 – PROBABILITY AND QUEUEING THEORY

(Regulations – 2017)

Time : Three hours　　　　　　　　　　　　　　Maximum : 100 marks

(Statistical Tables to be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $P(A)=\frac{1}{3}, P(B)=\frac{3}{4}$, and $P(A\cup B)=\frac{11}{12}$. Find $P(A/B)$ and $P(B/A)$.

2. State Baye's theorem.

3. The joint pdf of the random variable (X,Y) is given by $f(x,y)=k\,xy\,e^{-(x^2+y^2)}, x>0, y>0$. Find the value of k.

4. In fitting a straight line $y=ax+b$, what is the formula to find the sum of the squares of the residuals?

5. Write the classification of random process.

6. If the tpm of a Markov chain is $\begin{pmatrix} 0 & 1 \\ ½ & ½ \end{pmatrix}$, find the steady-state distribution of the chain.

7. What do the letters in the symbolic representation $(a/b/c):(d/e)$ of a queuing model represent?

8. Write down the Little's formulas that hold good for the infinite capacity Poisson queue models.

9. Write down the average queue length in the model (M/M/s):(k/FIFO).

10. What is a Jackson network?

PART B — (5 × 16 = 80 marks)

11. (a) (i) A random variable X has the following probability distribution: (8)

x:	-2	-1	0	1	2	3
$p(x)$:	0.1	k	0.2	2k	0.3	3k

Evaluate the following (i) K (ii) $P(-2<X<2)$ (iii) the cdf of X (iv) mean of X

(ii) The mileage which car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000 km. Find the probabilities that one of these tires will last (i) at least 20,000 km and (ii) at most 30,000 km. (8)

Or

(b) (i) The number of breakdowns of a computer is an RV having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month.

(1) without a breakdown (3)

(2) with only one breakdown (3)

(3) with at least one breakdown (2)

(ii) The life of a semiconductor laser at a constant power is normally distributed with a mean of 7000 hours and a standard deviation of 600 hours.

(1) What is the probability that a laser fails before 5000 hours? (4)

(2) If three lasers are used in a product and they are assumed to fail independently, what is the probability that all three are still operating after 7000 hours? (4)

12. (a) (i) Determine the value of C for the joint probability mass function of X and Y is given by $f(x,y) = C(x+y), x = 1,2,3$ and $y = 1,2,3$. Also, determine the following:

(1) Find the marginal probability distributions of X and Y. (3)

(2) Conditional probability distribution of X given that $Y = 2$. (3)

(3) Are X and Y independent? (2)

(ii) Compute the coefficients of correlation between X and Y using the following data. (8)

X	1	3	5	7	8	10
Y	8	12	15	17	18	20

Or

(b) (i) Determine the value of c such that the function $f(x,y) = cxy$, for $0 < x < 3$ and $0 < y < 3$ satisfies the properties of a joint probability density function. Also, find the covariance between X and Y. (8)

(ii) If the joint probability density function of (X, Y) is given by $f(x,y) = x + y, 0 \leq x, y \leq 1$, find the probability density function of $U = XY$. (8)

13. (a) (i) The Probability distribution of the process $\{X(t)\}$ is given by

$$P(X(t)=n)=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, n=1,2,3,\ldots\\ \dfrac{at}{1+at}, n=0\end{cases}$$

Show that it is not stationary. (8)

(ii) Prove that the difference of two independent Poisson processes is not a Poisson process. (8)

Or

(b) (i) Show that the random process $X(t)=A\cos(\omega t+\theta)$ is stationary if A and ω are constant and θ is uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) The transition probability matrix of a Markov chain $\{X_n\}, n = 1,2,3\ldots$ having 3 states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7, 0.2, 0.1)$. Find $P(X_2 = 3)$ and $P(X_2 = 3, X_1 = 3, X_0 = 2)$. (8)

14. (a) (i) Derive the balanced equation of the birth and death process. (8)

(ii) There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour.

(1) What fraction of the time all the typist will be busy? (4)

(2) What is the average time a letter has to spend for waiting and for being typed? (4)

Or

(b) (i) Arrivals at a telephone booth are considered to be Poisson with an average time of 12 min, between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes.

(1) Find the average number of persons waiting in the system. (2)

(2) What is the probability that a person arriving at the booth will have to wait in the queue? (2)

(3) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call? (2)

(4) Estimate the fraction of the day when the phone will be in use. (2)

(ii) A Supermarket has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in Poisson fashion at the rate of 10 per hour,

(1) What is the probability that a customer has to wait for service? (3)

(2) What is the expected percentage of idle time for each girl? (2)

(3) If the customer has to wait in the queue, what is the expected length of his waiting time? (3)

15. (a) (i) Derive the Pollaczek-Khinchine formula for Non-Markovian queuing model M/G/1. (8)

(ii) The Grabeur-Money Savings and Loan has a drive-up window. During the busy periods for drive-up service, customers arrive according to a Poisson distribution with a mean of 16/h. From observations on the teller's performance, the mean service time is estimated to be 2.5 minutes, with a standard deviation of $\frac{5}{4}$ minutes. It is thought that the Erlang would be a reasonable assumption for the distribution of the teller's service time. Also, since the building (and drive-up window) is located in a large shopping center, there is virtually no limit on the number of vehicles that can wait. The company officials wish to know, on average, how long a customer must wait until reaching the window for service, and how many vehicles are waiting for service? (8)

Or

(b) (i) A one-man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop? Also find the average time a customer must wait for service. (8)

(ii) The Drive-It-Through-Yourself car wash decides to change its operating procedure. It installs new machinery that permits the washing of two cars at once (and one if no other cars wait). A car that arrives while a single car is being washed joins the wash and finishes with the first car. There is no waiting capacity limitation. Arrivals are Poisson with mean 20/hour. The time to wash a car is exponentially distributed with a mean of 5 minutes. What is the average line length? (8)